IN SEARCH OF THE DIALECTIC ON THE DIVIDED LINE
(PLATO, *REPUBLIC* VI 509 d - 510 a)[1]

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This paper is intended to provide an explanation of the much-discussed passage 509 d - 510 a of Plato's *Republic*, in accordance with the context and his overall philosophy. The main aim is to refute erroneous interpretations, which twist the true meaning of his words, since they lead to impasses and misconstructions. I seek the mathematical ratios by which the line is twice intersected, since only if the reasons are identified can we speak of dialectic, which is mainly realised, not by division *κατά μέρος* ("by part") and chance, but *κατ᾽ εἶδος* ("by kind"), by certain ratios (*R.*, 534 a 5-8), i.e. the natural section of the line, as it is realised in the *Sophist*, *Statesman* and *Phaedrus*. Hypothesis as a component of dialectic is present in the participle *λαβών* ("having taken"), which is time-conditional and means "if or when you take". The object of "*λαβών*" is *διττὰ εἴδη... ὥσπερ γραμμὴν* ("two kinds... as a line"), i.e. the universe (*τὸ πᾶν*), and not "*τὰ τμήματα*" ("the segments"). It even-

tually proved impossible to discover the ratios of the section of the line, and therefore dialectic. However, from all this effort I hope to set aside some misinterpretations, which have kept the gate of Plato's wonderful intellectual edifice hermetically shut for centuries. The benefit of this failed attempt is that we become διαλεκτικώτεροι ("more skilled in argument", *Plt.*, 285 d). It must, of course, be stated from the outset that parts of the interpretation which I am attempting to prove as the only correct one, have already been supported at various times by eminent Classical philologists and philosophers, without, unfortunately, being accepted. Thus explanations have prevailed which are unconnected to the actual meaning of Plato's words, since the meaning is radically warped. The main points of the interpretation attempted here are the following.

1. The correct reading of the phrase: ὥσπερ τοίνυν γραμμὴν δίχα τετμημένην λαβὼν ἄνισα τμήματα ("having taken it as a line cut into two unequal segments") is ἀνὰ ἴσα ("proportional") and not ἄνισα ("unequal"). The reading ἴσα or ἀνὰ or ἄν ἴσα is given by Codex F and adopted by, among others, Astius, Stallbaum, Richter, Duemmler, Iamblichus and Plato's friend, the mathematician Archytas. The reading ἄνισα comes up against insuperable problems from a linguistic and mathematical point of view. These are that the adverb δίχα generally, and especially when used in a mathematical sense, as here, always implies division into two equal parts (cf. Arist *Top.* VI 4, 142 b 11-19), either κατὰ λόγον ("by ratio") or κατ᾽ ἀριθμόν ("by number"). Serious difficulties also arise from a mathematical point of view, since, if the segments are unequal, then they are not necessarily symmetrical (*Prm.*, 140 b-c); in this case they would be ἄλογα, i.e. there would be no ratio (in integers). The word ἴσα here means ἴσα κατὰ λόγον, i.e. proportional, but they are ἄνισα κατ᾽ ἀριθμόν, i.e. arithmetically unequal. This double meaning of the word ἴσα is found, for instance, in the *Laws* (V 744 c, VI 757c): ὡς ἰσαίτατα τῷ ἀνίσῳ συμμέτρῳ "so that they are most equal by a rule of symmetrical inequality" and in *Gorgias* 508 a. The preposition ἀνὰ is necessary, since the line is cut twice into two equal parts (δὶς δίχα), once from a gnosiological and once from an ontological point of view. Only under this precondition was Plato able to form the proportion (534 a): οὐσία: γένεσις = νόησις: δόξα (essence: generation: intellection: opinion). Ἀνὰ ἴσα means that the four

segments of the first section are proportional to each other. The issue is to discover the axiological ratio among them.

2. The phrase: πάλιν τέμνε ἑκάτερον τὸ τμῆμα ἀνὰ τὸν αὐτὸν λό-γον ("cut each segment again in the same ratio"), does not mean that each of the two segments of the line will be cut by the same ratio as the previous section, as everyone except the ancient commentators Askle-pios (*Arist. In Met.-CAG* VI Kroll,142), Archytas and Iamblichus (*De comm.math. scientia*,36,38) has wrongly believed to this day. If this were the meaning of Plato's words, he should have said κατὰ τὸν αὐτὸν λόγον, "by the same ratio", or, better, κατὰ τὴν αὐτὴν ἀναλογίαν, "by the same proportion". The phrase ἀνὰ τὸν αὐτὸν λόγον simply implies that the section of the two segments arising from the first division will be done by the same ratio, so that all four segments will be proportional to each other, exactly as with the first section (ἀνὰ ἴσα). Here the same action is repeated. The word-group ὁ αὐτὸς λόγος ("the same ratio") – as in Gorgias (*Encomium of Helen* 14), Aristotle (*Top.* VIII 158 b 33-35, *Eth.Nic.* V 3, 1181 b 4-5) , Plato (*Tim.,* 32 b7, *Phaid.,*110 b5-6 and Euclid (*Elements* V, Defin. 6) –is a technical term denoting proportion. Thus there is no reference to the relation between the two ratios, that of the first section and the following two.

The difference in meaning between the two interpretations is due to the use of the prepositions ἀνὰ and κατά. As the passage has been interpreted to date, it is presupposed that Plato wrote: πάλιν τέμνε ἑκάτερον τὸ τμῆμα **κατὰ** τὸν αὐτὸν λόγον. The meaning, according to this reading, is that the ratio of the section of each of the divided seg-ments is the same as the ratio of the first section of the whole line. But it is not specified whether the ratio of the second section is the same as that of the third of the two divided segments. The fact that these ratios are the same is not self-evident but emerges from the follow-ing Euclidean theorem (*Elements* V, 11): "The ratios, which are equal to the same ratio, are also equal to each other". On the contrary, the phrase **ἀνὰ** τὸν αὐτὸν λόγον means that the ratio of the second section is the same as that of the third, i.e. there is a proportion between the four segments, while there is no reference to the relation of these two ratios to the ratio of the original section, which is obviously not the same but different. Studies by mathematicians in collaboration with myself have produced the following conclusion: in order for there to be

an increasing gradation of the four segments, e.g. a>b>c>d - as required due to their correspondence to the states of mind and to the reality (511 d -e) -, the ratio of the first section must be greater than that of the following sections. This mathematical observation can reasonably be taken to express the idea that, as regards value, the difference between the faculties of the soul and the corresponding beings which arise from the first division, specifically the difference between intellection and opinion, essence and generation, is greater than that between these and the subdivisions, i.e. ἐπιστήμη and διάνοια, πίστις and εἰκασία (intelligence, and reason, belief and conjecture), as well as the corresponding beings, i.e. between ideas and mathematics, material objects and their images. The result of the dominant interpretation is the paradoxical and nonsensical fact that the two middle segments are equal κατ᾽ ἀριθμόν (Euclid, *Elements*, V, 7, 9).

3. As regards passage 510 a 8-10, Ἦ καὶ... ὡμοιώθη, I support the interpretation of those who believe that the pronoun αὐτὸ refers to the whole line, i.e. the entire world, both intelligible and visible, and that the images and the example belong to the intelligible (example) and visible (image) world respectively, rather than the sensible things and their likenesses.

4. a) The upper part of the line may equally well be the shortest, as the Pythagorean Brontinus, Iamblichus and a few modern commentators believe. This is because - among other things - on analysis of the number by a random ratio, the unit or the smallest number corresponds to the Good, which is the One and placed at the top of the line. The many, on the contrary, are connected to the material of the visible world. Thus the largest segment must be the lowest. Everything depends on the signification of the line. If it is evaluated negatively, as obscurity (ἀσάφεια), the upper part must be the smallest, because the lesser evil is considered good (Arist. *EN.* E1, 1129 b 8). If, on the contrary, it is allocated a positive value, as clarity (σαφήνεια), then the upper part is the largest, because it is of greater value.

b) No section can be made by the ἄκρον καὶ μέσον λόγον ("extreme and mean ratio"), i.e. by geometrical proportion in the modern sense. In other words, we can have no χρυσῆ τομή ("golden section"), because the two segments, into which the line is thus divided, will be ἀσύμμετρα, ἀσύμφωνα and ἄλογα ("asymmetrical, not in accordance and

disproportional"), given that the division is by ratios of integers (Euclid, *Elements* XIII, theorem 6, and VI, theorem 30).

c) The proportion which Plato refers to as the result of the three sections by different ratios, noesis : doxa = episteme : pistis = dianoia : eikasia (intellection : opinion = intelligence : belief = reason : conjecture), is justified because the divided segments are contained in the undivided in a continuous line, based on certain attributes of proportions regarding the composition and division of ratios and magnitudes (Euclid, *Elements* V, Defin. 14, 15 and V theorems 17 and 18).

d) The ratios arising from the successive sections may be represented by musical notes based on Pythagorean music*.
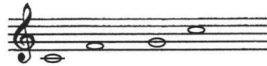

e) In the present study, ratios up to ten have been understood as numbers used by Plato, for instance: 12 (division of the ideal state), 729 (ethos of statesmen), 46656 (number of Timaeus), 5040 (number of inhabitants of the ideal state), 760000 (geometrical number), 792 (multiple of 36, i.e. the Pythagorean tetraktys).

f) Finally, I believe it very probable that the first ratio is 3 and the second 2, i.e. those of the double and triple interval on the model of *Timaeus* (36 a-b), the *Statesman* (266 a) and the *Republic* (IX 587 c-e), because - starting from the unit - the seventh number formed by the double and triple ratio, or multiples, is simultaneously a square and a cube, such as $64 (= 8^2$ or $4^3)$ and $729 (= 27^2$ or $9^3)$, which represent corporeal and incorporeal substance (Philo. jud., *On the Account of the World's Creation Given by Moses*, 30).

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* The musical composition is made by Prof. E. Moutsopoulos - Member of the Academy of Athens. The distances are: episteme = 1, dianoia = 2, pistis = 3, eikasia = 6, or in reverse order: episteme = 6, dianoia = 3, pistis = 2, eikasia = 1.

The image of the line

Good = One

Evil = Many
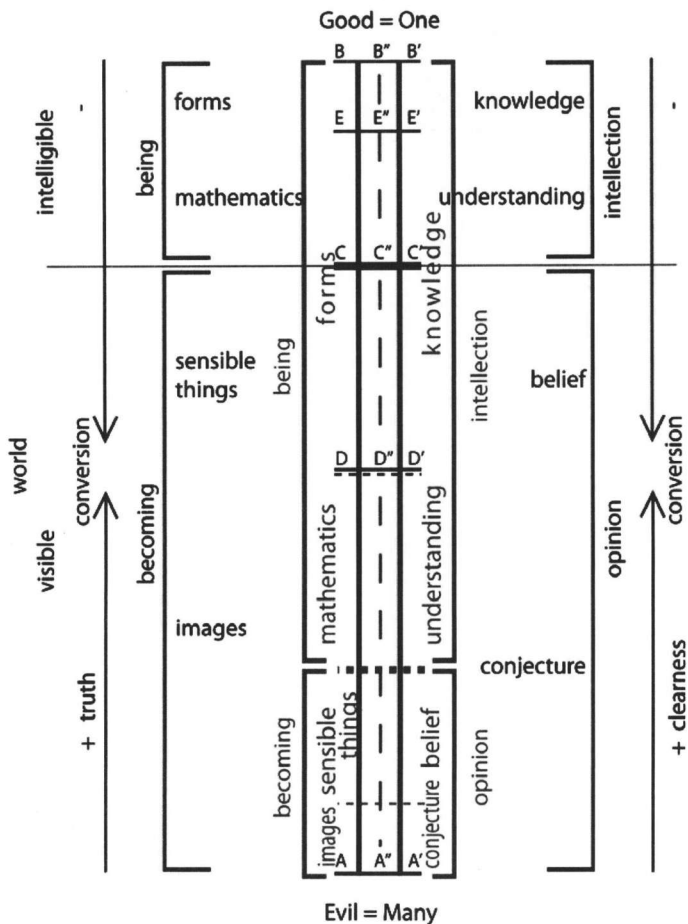
AB= ontology, A'B'= epistemology, A"B"= theory of values

Let (AB)=the whole (Figure)

From the Figure let $(BC) < (CA)$ and $(CA)=\xi \cdot (BC)$

Also $(B'C')<(C'A')$ and $(C'A')= \xi \cdot (B'C')$ }   (1)

where ξ= a rational number greater than 1

From the Figure $1<\dfrac{(AD)}{(DC)} = \dfrac{(CE)}{(EB)} < \xi$ (2)

Then $\dfrac{(AD)}{(DC)} = \dfrac{(CE)}{(EB)} \Rightarrow \dfrac{(AD)}{(CE)} = \dfrac{(DC)}{(EB)} = \dfrac{(AD)+(DC)}{(CE)+(EB)} = \dfrac{(AC)}{(CB)} < \xi$ (3)

(3) implies that :

$(DC)=\xi \cdot (EB)$ and due to (2) $(DC) > (CE)$ (4)

(2) and (4) gives us:

$$(AD) > (DC) > (CE) > (EB)$$ (5)

as required according to Plato.

It follows that for all the values of the ratio (2) in the interval $(1 , \xi)$ relation (5) applies.

If we suppose that there is obscurity and accept that the first ratio

$\dfrac{(AC)}{(BC)}=\xi$

is equal (i.e. the same as) to the second ratio $\dfrac{(AD)}{(DC)} = \dfrac{(CE)}{(EB)} = \xi$ (6)

then, since (3) implies that $(AD) = \xi \cdot (CE)$
and (6) implies that $(AD) = \xi \cdot (DC)$
it emerges that $(CE) = (DC)$ (7)
a relation which is   inconsistent with Plato's initial consideration (5)

From the above we conclude that Plato meant that these two ratios are different from each other

We name the second ratio $\dfrac{(AD)}{(DC)} = \dfrac{(CE)}{(EB)} = \xi'$

We will prove that the first ratio $\left(\dfrac{(AC)}{(BC)} = \xi \right)$ (8)

is greater than the second ratio $\left(\dfrac{(AD)}{(DC)} = \dfrac{(CE)}{(EB)} = \xi' \right)$ (9)

(9) implies: $\dfrac{(DC)}{(AD)} = \dfrac{1}{\xi'} \Rightarrow \dfrac{(DC)}{(AD)+(DC)} = \dfrac{1}{\xi'+1} \Rightarrow \dfrac{(DC)}{(AC)} = \dfrac{1}{\xi'+1} \Rightarrow$

$$(DC) = \frac{(AC)}{\xi'+1}$$ (10)

(9) implies: $\dfrac{(CE)}{(EB)} = \xi' \Rightarrow \dfrac{(CE)}{(CE)+(EB)} = \dfrac{\xi'}{\xi'+1} \Rightarrow \dfrac{(CE)}{(CB)} = \dfrac{\xi'}{\xi'+1} \Rightarrow$

$$(CE) = \frac{\xi'}{\xi'+1} \cdot (CB)$$ (11)

But according to Plato's $(CD) > (CE)$.

Replacing (CD) and (CE) from (10) and (11) we have

$$\frac{(AC)}{\xi'+1} > \frac{\xi' \cdot (CB)}{\xi'+1} \Rightarrow (AC) > \xi' \cdot (CB) \Rightarrow \frac{(AC)}{(CB)} > \xi'$$

and replacing from (8) we have $\xi > \xi'$.

We have therefore proven that the first ratio $\dfrac{(AC)}{(BC)}$

πέντε κυρίων
is greater than the second ratio $\dfrac{(AD)}{(DC)} = \dfrac{(CE)}{(EB)}$.

ΠΕΡΙΛΗΨΗ

ΑΝΑΖΗΤΗΣΗ ΤΗΣ ΔΙΑΛΕΚΤΙΚΗΣ ΣΤΗΝ ΔΙΗΡΗΜΕΝΗ ΓΡΑΜΜΗ
(ΠΛΑΤ., *Πολιτ.* VI 509 d–510 a)

Στὴν μελέτη αὐτὴ παρουσιάζονται κατ' ἀρχὴν οἱ δυσκολίες κα-
τανόησης τοῦ ἐν λόγῳ χωρίου, ἂν δεχθῆ κάποιος τὴν ἐπικρατέστερη
ἑρμηνεία. Στὴν ἐκδοχὴ κατὰ τὴν ὁποία ὁ Πλάτων ἔγραψε «γραμμὴν
δίχα τετμημένην ἄνισα τμήματα» ἀναφύονται οἱ ἑξῆς δυσκολίες: 1)
τὸ ἐπίρρημα *δίχα* δὲν εἶναι συμβατὸ μὲ τὰ *ἄνισα τμήματα*, ἀφοῦ *δίχα*
τέμνω σημαίνει διαιρῶ σὲ δύο ἴσα μέρη· 2) ἡ φράση αὐτὴ ἀποτελεῖ
σολοικισμό, ἀφοῦ θὰ ἔπρεπε νὰ συνοδεύεται μὲ τὴν πρόθεση *εἰς*, δη-
λαδὴ *εἰς ἄνισα*· 3) ἂν τὰ τμήματα εἶναι ἄνισα, τότε ἐνδέχεται νὰ εἶναι
ἀσύμμετρα, ἀσύμφωνα, ἀνάρμοστα, ἄρρητα, ἄλογα καὶ ὄχι προσή-
γορα. Ἐπίσης ἂν δεχθοῦμε ὡς ἀκριβῆ τὴν ἑρμηνεία ὅτι ἡ φράση «*ἀνὰ*
τὸν αὐτὸν λόγον» σημαίνει ὅτι οἱ λόγοι εἶναι οἱ ἴδιοι, τότε ἀναπό-
φευκτα τὰ δύο μεσαῖα τμήματα θὰ εἶναι ἴσα, πρᾶγμα ποὺ ἀντιφά-
σκει πρὸς τὸ πνεῦμα τοῦ Πλάτωνος.

Ἡ λύση ποὺ προτείνεται ἔγκειται στὸ ὅτι ἡ ὀρθὴ γραφὴ εἶναι *ἀνὰ*
ἴσα (κατὰ λόγον καὶ ὄχι κατ' ἀριθμόν), δηλαδὴ ἀνάλογα. Ἡ γραμμὴ
τέμνεται δύο φορὲς δίχα, ὁπότε σχηματίζονται τέσσερα τμήματα
ἀνάλογα μεταξύ τους, δηλαδή: οὐσία: γένεσις = νόησις: δόξα. Ὁ λό-
γος αὐτὸς εἶναι διαφορετικὸς ἀπὸ ἐκεῖνον τῶν ἑπόμενων τομῶν. Ἡ
ἀναλογία ποὺ σχηματίζεται εἶναι: *ἐπιστήμη: διάνοια = πίστις: εἰκα-*
σία. Ἂν οἱ ὅροι τῆς ἀναλογίας αὐτῆς τεθοῦν ἐναλλάξ, τότε ὁδηγού-
μαστε στὸν πρῶτο λόγο: *ἐπιστήμη: πίστις = διάνοια: εἰκασία = νόη-*
σις: δόξα, ἐπειδὴ ἡ νόηση εἶναι τὸ ἄθροισμα τῆς ἐπιστήμης καὶ τῆς
διανοίας, ἐνῶ ἡ δόξα, τῆς πίστεως καὶ τῆς εἰκασίας. Ἂν στὴν γραμμὴ
ἀποδίδεται ἀρνητικὴ σημασία (ἀσάφεια, μὴ ἀλήθεια), τότε τὸ ἄνω
μέρος θὰ πρέπει νὰ εἶναι τὸ μικρότερο, ἐνῶ στὴν ἀντίθετη περίπτωση
(σαφήνεια, ἀλήθεια) θὰ εἶναι τὸ μεγαλύτερο. Καὶ οἱ δύο δυνατότη-
τες εἶναι ἀναγκαῖο νὰ χρησιμοποιηθοῦν διαδοχικὰ γιὰ τὴν μόρφωση
τοῦ ἀνθρώπου. Ἡ ἀμάθεια ὁδηγεῖ τὸν ἄνθρωπο στὴν κακία καὶ τὸ
σκότος. Μὲ τὴν παιδεία ὅμως ἐπέρχεται μεταστροφὴ ἀπὸ τὸ σκότος
στὴν ἀλήθεια καὶ τὸ φῶς.

Εἶναι ἐδῶ ἀξιοσημείωτο τὸ γεγονὸς ὅτι μὲ τὴν διαίρεση τῆς
γραμμῆς σὲ τέσσερα τμήματα συντελεῖται καὶ ἡ ἀπεικόνιση τῶν

κλάδων τῆς φιλοσοφίας: ὀντολογίας, γνωσιολογίας, ἀξιολογίας, τῆς ἠθικῆς καὶ τῆς πολιτικῆς φιλοσοφίας. Ἡ τελευταία δικαιολογεῖται ἀπὸ τὸ ὅτι στὸ σημεῖο αὐτὸ γίνεται πραγμάτευση τοῦ ἀγαθοῦ. Τὸ σύνολο ὅλων αὐτῶν, δηλαδὴ ἡ γραμμὴ ὡς ὅλον, ἀντικατοπτρίζει τὴν πολιτική, ἀφοῦ τὸ ἀγαθὸν καὶ ἡ εὐδαιμονία ὅλων τῶν πολιτῶν πρέπει νὰ εἶναι τὰ κύρια ἀντικείμενα ἐνασχόλησης τοῦ ἀληθινοῦ πολιτικοῦ. Ὁ ἀριθμὸς πέντε συμπίπτει μὲ τοὺς πέντε παράγοντες τῆς γνώσης τῶν ὄντων: ὄνομα, λόγος εἴδωλον, ἐπιστήμη καὶ ὄντως ὂν (Ζ΄ ἐπ. 342 a-e). Ἄλλωστε κατὰ τὸν Ἀριστοτέλη ὁ σκοπὸς (τέλος) τῆς πολιτικῆς περιέχει τοὺς σκοποὺς ὅλων τῶν ἄλλων ἐπιστημῶν (ΗΝ Α2, 1194a6). Οἱ λόγοι τῶν τριῶν αὐτῶν τομῶν συγκροτοῦν μουσικὴ σύμφωνα μὲ τοὺς Πυθαγορείους. Ὁ δὲ Πλάτων ὀνομάζει τὴν φιλοσοφία στὸν Φαίδωνα ὡς «μεγίστην μουσικήν». Μὲ τὶς τομὲς αὐτὲς γίνεται διαίρεση τῶν ἐναντίων, ἀφοῦ οἱ ἰσότητες εἶναι ταυτοχρόνως καὶ ἐναντιότητες, ὅπως τὸ παράδειγμα καὶ τὸ εἴδωλον.

Τὸ γεγονὸς αὐτὸ παραπέμπει εὐθέως στὴν ὕπαρξη διαλεκτικῆς στὸ ὑπὸ ἐξέταση χωρίο. Τὸ μεγάλο ζητούμενο εἶναι ν' ἀνευρεθοῦν οἱ δύο λόγοι τῶν τριῶν τομῶν τῆς γραμμῆς, ὥστε αὐτή, ἀπὸ δυνάμει ποὺ εἶναι, νὰ γίνη ἐνεργείᾳ, νὰ λάβη δηλαδὴ σάρκα καὶ ὀστᾶ.

Αὐτά, δηλαδὴ οἱ λόγοι οἱ ὁποῖοι σὲ σχῆμα ἀναλογίας συνθέτουν τὸ ἕν (Τιμ., 30 c), δηλαδὴ τὸ ἀγαθόν, δὲν ἀποτελοῦσαν ἑπτασφράγιστο μυστικὸ (ἀπόρρητα), ἀλλὰ μᾶλλον δὲν μποροῦσαν νὰ εἰπωθοῦν ἐκ τῶν προτέρων (ἀπόρρητα), πρὶν δηλαδὴ τὰ ἀνακαλύψει ὁ ἀναγνώστης ἢ ὁ ἀκροατὴς (Νόμ., ΙΒ 968 d) μὲ τὴν μαιευτικὴ μέθοδο (Θεαίτ., 210 b-c). Τοῦτο συμβαίνει, ἐπειδὴ θὰ ἦταν ἀνάρμοστο καὶ ἀπρεπές (ἀναρμοστία καὶ ἀπρέπεια) νὰ λεχθοῦν ὅπως λέγονται τὰ ἄλλα μαθήματα (οὐ ῥητόν. Ζ Ἐπ., 344 d, 341 c). Ἔτσι οἱ ἀρχὲς, δηλαδὴ τὸ ἓν καὶ τὸ ἀγαθόν, ποὺ ἔχουν μεγαλύτερη ἀξία (τιμιώτερα, Φαῖδρ., 278 d) λέγονται ὑπὸ μορφὴν αἰνίγματος, ἡ λύση τοῦ ὁποίου πρόκειται ὡς ἆθλον στὸν συνομιλητὴ (Τίμ., 54 d), ὅπως ἀκριβῶς ἡ ἐλευθερία στοὺς δούλους (Ἀριστ., Πολ., Η 10,1330 a 33). Ὅλα φαίνεται ὅτι συγκλίνουν στὴν ὑπόθεση ὅτι ὁ τρόπος αὐτὸς ἐκφορᾶς τῆς φιλοσοφίας τοῦ Πλάτωνος ἀποτελεῖ τὴν λεγόμενη ἄγραφη διδασκαλία του. Σχετικὰ μὲ τὸ θέμα αὐτὸ πολὺ σημαντικὴ μπορεῖ νὰ θεωρηθῆ ἡ μαρτυρία τοῦ Νικομάχου τοῦ Γερασηνοῦ (Ἀριθμ. Εἰσαγωγὴ Ι γ), ὅτι τὸ 13 βιβλίο τῶν Νόμων, δηλαδὴ ἡ Ἐπινομίς, εἶναι κατὰ τὴν

ἄποψη ὁρισμένων ὁ *Φιλόσοφος*, τὸ ἔργο δηλαδή, τὸ ὁποῖο προανήγγειλε ὁ Πλάτων στὸν *Σοφιστή* (217a) καὶ στὸν *Πολιτικό* (257a) ὅτι θὰ ἀπεργαζόταν. Ἀναφέρει μάλιστα τὰ βασικὰ σημεῖα τοῦ διαλόγου αὐτοῦ, ὅτι δηλαδὴ κύριο μέλημα τοῦ φιλοσόφου εἶναι νὰ βλέπη πρὸς τὸ ἕν, ποὺ ἀποτελεῖ τὴν προϋπόθεση κάθε γνώσης, εὐδαιμονίας καὶ μακαριότητας. Ἡ πορεία δὲν εἶναι ἄλλη ἀπὸ τὴν διαλεκτική (991b-992e).