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OVERPOPULATION, THE IDEAL CITY, AND PLATO'S MATHEMATICS

I. Introduction

The plethora of recent publications dealing with the population explosion, the maldistribution of the population, utopian cities, and family planning has reminded me of one of Plato's (427—347 B. C.) suggestions regarding city planning. According to the Athenian philosopher, the ideal city must consist of 5,040 men and their respective families (*Laws*, 737e—738b).

But why 5,040 ? Why not, for example, 5,000 or 6,000 households ?

Plato himself explains that 5,040 is desirable because it has many divisors (59), including the numbers 1 to 10 (*ibid.*).

I have always considered this explanation somewhat incomplete or secondary. After all, the chosen size for the ideal city is numerically «strange», and the 59 divisors do not appear to solve any of the problems discussed by Plato so profoundly even if his metaphysical assumptions and mystical doctrines were to be accepted unquestioningly. In fact, Aristotle himself, in his analysis of population problems and birth control, criticizes his great master rather vehemently (*Politics*, 1264b—1265b).

My theory is that Plato initially based his choice on his well—known belief in the *cosmic properties of the number seven*, as well as on the fact that 5,040 happens to be the value of *factorial seven*—it seems doubtful that this was merely coincidental ! Indeed, being the second greatest Pythagorean, he found numbers and their mystical qualities more than fascinating (Panos D. Bardis, «Symmetrical Consonance of Play, Rhythm, and Harmony : An Essay on Plato's Mathematics», *School Science and Mathematics*, January 1963, pp. 52—67). Besides, he and his pupils explored numbers so zealously and fruitfully that some conception of «factorials», long before Euler and Kramp, cannot be regarded as impossible in Plato's Academy. Moreover, the number seven was considered exceedingly sacred in ancient Greece. Finally, Plato himself was born on the *seventh* day of Thargelion, or May (Plutarch, *Table—Talk*, 717b).

II. Factorials

Once more, Plato did not deal with factorials in modern mathematical fashion. And the factorial symbols themselves are quite recent.

It was only in 1842, for instance, that Augustus de Morgan wrote an in-

teresting article, « Symbols», for the *Penny Cyclopaedia*, in which he complained that it was a «barbarism» to borrow the notation n ! from Germany.

In 1831, Thomas Jarrett of the University of Cambridge published *An Essay on Algebraic Development*, which included this equation : $a = a\,(a - 1)\,(a - 2) \ldots . 2 . 1.$

In Portugal, in 1824, Francisco Stocker completed his *Methodo Inverso dos Limites*, where he employed $1^{m}/_{1}$, a symbol which he had obviously taken from Kramp.

Christian Kramp of Strasbourg used this symbol in 1808 in his *Éléments d'Arithmétique Universelle*, which also introduced what its author called «*la notation très simple n !*»

In the same year, Adrien—Marie Legendre, the French mathematician who simplified Euclid's geometry, wrote a short paper containing the factorial symbol $\Gamma\,(n + 1)$. Three years later, he included this notation in his *Exercises de Calcul Intégral*.

A star had been employed by Johann Bernhard Basedow, in his *Bewiesene Grundsätze der reinen Mathematic* (1774), thus : $5^{*} = 5.4.3.2.1.$

Two years earlier, the French mathematician Alexis Vandermonde wrote the following in the *Histoire de l'Académie* : «*Je represénte par* $[p]^{n}$ *le produit de n facteurs*».

Finally, in 1751, the Swiss physicist and mathematician Leonhard Euler published a short paper entitled «*Calcul de la Probabilité*», where he spoke of a «*nombre de cas* 1.2.3.4 . . . . *m etant posé pour abréger* $= M$».

III. The Sacred Seven

The mystical and magical qualities of the number seven are mentioned in myriads of sources throughout the world. A few examples should suffice here.

In India, the *Rig—Veda* of the second millennium B. C. refers to seven as a sacred number very frequently. In the same country, the symbol for seven was a stylized drawing of the human head—because of its *seven* apertures.

Similar convictions prevailed among the Babylonians almost 5,000 years ago. Such convictions were also associated with the *seven* «planets», namely, the Sun, the Moon, Mercury, Venus, Mars, Jupiter, and Saturn, with which the Babylonian astrologers were familiar. Later on, this led to the development of the seven-day week, which the Jews adopted and finally brought to Rome in the first century A. D. We thus have the number seven incorporated into various words meaning «week» : Greek *hebdomas*, medieval Latin *septimana*, Italian *settimana*, Spanish *semana*, French *semain*, and so forth.

The Bible itself is replete with references to the number seven. For instance, according to Revelation 5 : 5, «the Root of David, hath prevailed to open the book, and to loose the seven seals thereof». Incidentally, at about the same time

as Saint John the Divine wrote his Apocalypse, the Mahayanists of South India produced an 8,000—line work, *The Perfection of Wisdom*, one of their greatest treatises, which was called a book «sealed with seven seals».

In Plato's own country, where Oriental influences were anything but uncommon, countless masterpieces presented the number seven as sacred. Homer, for example, wrote that «Zeus brought the seventh day» (*Odyssey*, XV, 477). Hesiod added that «the seventh day is holy» (*Works and Days*, 770). And Aeschylus referred to Apollo as the «Commander of Sevens» (*Seven Against Thebes*, 800). It is further significant that, like Plato, Apollo was born on the *seventh* day of Thargelion, which was thus sacred to the god of the sun.

IV. Conclusion

It seems that the divisor concept concerning the size of Plato's ideal city is incomplete and secondary. Instead, innumerable data indicate that the Athenian philosopher was originally inspired by the cosmic qualities of the number seven and the fact that 5,040 is the value of factorial seven.

(I will be happy to hear from readers who have additional information supporting or invalidating my theory).

P. S. Incidentally, if Plato's only intention had been to have the lowest number containing divisors 1 to 10, then he would have chosen 2,520, which would have made Aristotle less critical. This lower figure may be expressed thus: 1.2.2.2.3.3.5.7. (Aristotle always rounded Plato's number to 5,000—e.g., twice in *Politics*, 1265a).

ΠΟΙΗΣΙΣ ΕΙΣ ΑΡΧΑΙΟΥΣ ΣΤΙΧΟΥΣ

Εὐδαιμονίζω σ' ὧν ἔχεις σκηνωμάτων,
'Ιάκωβε· ναίεις πάγκαλ', 'Ισραὴλ σταθμά·
ἐκτείνεται δ' ὅμοια βαθυκόλπῳ νάπῃ
κήπου τ' ἐπὶ ῥοαῖσιν ἀνθοῦντος γάνει.

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